

# Capacity of the Gaussian Two-Hop Full-Duplex Relay Channel with Self-Interference

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## Abstract

In this paper, we investigate the capacity of the Gaussian two-hop full-duplex (FD) relay channel with self-interference. This channel is comprised of a source, an FD relay, and a destination, where a direct source-destination link does not exist and the FD relay is impaired by self-interference. We model the self-interference as an additive Gaussian random variable whose variance is proportional to the amplitude of the transmit symbol at the relay. For this channel, we derive the capacity and propose an explicit capacity-achieving coding scheme. Thereby, we show that the optimal input distribution at the source is Gaussian and its variance depends on the amplitude of the transmit symbol at the relay. On the other hand, the optimal input distribution at the relay is discrete or Gaussian, where the latter case occurs only when the relay-destination link is the bottleneck link. The derived capacity converges to the capacity of the two-hop ideal FD relay channel without self-interference and to the capacity of the two-hop half-duplex (HD) relay channel in the limiting cases when the self-interference is zero and infinite, respectively. Our numerical results show that significant performance gains are achieved using the proposed capacity-achieving coding scheme compared to the achievable rates of conventional FD relaying and HD relaying.

## I. INTRODUCTION

In wireless communications, relays are employed in order to increase the data rate between a source and a destination. The resulting three-node channel is known as the relay channel [2]. If the distance between the source and the destination is very large or there is heavy blockage, then the relay channel can be modeled without a source-destination link, which is known as the two-hop relay channel. For the relay channel, there are two different modes of operation for the relay, namely, the full-duplex (FD) mode and the half-duplex (HD) mode. In the FD mode, the relay transmits and receives at the same time and in the same frequency band. As a result, FD relays are impaired by self-interference, which is the interference caused by the relay's transmit signal to the

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relay's received signal. Latest advances in hardware design have shown that the self-interference of an FD node can be suppressed by about 120 dB, see [3]. This has led to an enormous interest in FD communication [3]-[25]. On the other hand, in the HD mode, the relay transmits and receives in the same frequency band but in different time slots or in the same time slot but in different frequency bands. As a result, HD relays completely avoid self-interference. However, since an HD relay transmits and receives only in half of the time/frequency resources compared to an FD relay, the achievable rate of the two-hop HD relay channel may be significantly lower than that of the two-hop FD relay channel.

Information-theoretic analyses of the capacity of the two-hop HD relay channel were provided in [26], [27]. Thereby, it was shown that the capacity of the two-hop HD relay channel is achieved when the HD relay switches between reception and transmission in a symbol-by-symbol manner and not in a codeword-by-codeword manner, as in conventional HD relaying [28]. Moreover, in order to achieve the capacity, the HD relay has to encode information into the silent symbol intervals created when the relay receives [27]. For the Gaussian two-hop HD relay channel, it was shown in [27] that the optimal input distribution at the relay is discrete and includes the zero (i.e., silent) symbol. On the other hand, the source transmits using a Gaussian input distribution when the relay transmits the zero (i.e., silent) symbol and is silent otherwise.

The capacity of the Gaussian two-hop FD relay channel with ideal FD relaying without self-interference was derived in [2]. However, in practice, canceling the self-interference completely is not possible due to limitations in channel estimation precision and imperfections in the transceiver design [8]. As a result, self-interference has to be taken into account when investigating the capacity of the two-hop FD relay channel. Despite the considerable body of work on FD communication, see [3]-[25] and references therein, the capacity of the two-hop FD relay channel with self-interference has not been explicitly characterized. Therefore, in this work, we study the capacity of the Gaussian two-hop FD relay channel with self-interference. In particular, we assume that the source-relay and relay-destination links are additive white Gaussian noise (AWGN) channels. Moreover, we model the self-interference as a conditionally Gaussian distributed random variable (RV) whose variance is dependent on the amplitude of the symbol transmitted at the relay. For this relay channel, we derive the capacity and propose an explicit coding scheme which achieves the capacity. Thereby, we show that the capacity-achieving input distribution at the source is Gaussian and its variance depends on the amplitude of the symbol transmitted at the relay. Moreover, the source transmits only when the amplitude of the symbol transmitted by the relay is smaller than a threshold, otherwise, the source is silent. On the other hand, the optimal input distribution at the relay is

discrete or Gaussian, where the latter case occurs only when the relay-destination link is the bottleneck link. The derived capacity converges to the capacity of the two-hop ideal FD relay channel without self-interference [2] and to the capacity of the two-hop HD relay channel [27] in the limiting cases when the self-interference is zero and infinite, respectively. Our numerical results show that significant performance gains are achieved with the proposed capacity-achieving coding scheme compared to the achievable rates of conventional FD relaying and HD relaying.

This paper is organized as follows. In Section II, we present the models for the channel and the self-interference. In Section III, we present the capacity of the considered channel and propose an explicit capacity-achieving coding scheme. Numerical examples are provided in Section IV, and Section V concludes the paper.

## II. SYSTEM MODEL

In the following, we introduce the models for the two-hop FD relay channel and the self-interference.

### A. Channel Model

We assume a two-hop FD relay channel comprised of a source, an FD relay, and a destination, where a direct source-destination link does not exist. We assume that the source-relay and the relay-destination links are AWGN channels, and that the FD relay is impaired by self-interference. In symbol interval  $i$ , let  $X_1[i]$  and  $X_2[i]$  denote RVs which model the transmit symbols at the source and the relay, respectively, let  $\hat{Y}_1[i]$  and  $\hat{Y}_2[i]$  denote RVs which model the received symbols at the relay and the destination, respectively, and let  $\hat{N}_1[i]$  and  $\hat{N}_2[i]$  denote RVs which model the AWGNs at the relay and the destination, respectively. We assume that  $\hat{N}_1[i] \sim \mathcal{N}(0, \hat{\sigma}_1^2)$  and  $\hat{N}_2[i] \sim \mathcal{N}(0, \hat{\sigma}_2^2)$ ,  $\forall i$ , where  $\mathcal{N}(\mu, \sigma^2)$  denotes a Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$ . Moreover, let  $h_{SR}$  and  $h_{RD}$  denote the channel gains of the source-relay and relay-destination channels, respectively, which are assumed to be constant during all symbol intervals, i.e., fading<sup>1</sup> is not considered. In addition, let  $h_{RR}[i]$  denote the self-interference channel gain in symbol interval  $i$ . In contrast to the source-relay and relay-destination channel gains,  $h_{SR}$  and  $h_{RD}$ , which are constant during the transmission when fading is not present, the self-interference channel gain,  $h_{RR}[i]$ , varies even in the absence of fading, as will be explained in the following subsection.

<sup>1</sup>As customary for capacity analysis, see e.g. [29], as a first step we do not consider fading and assume real-valued channel inputs and outputs. The generalization to a complex-valued signal model is relatively straightforward [30]. On the other hand, the generalization to the case of fading may be more involved and presents an interesting topic for future research.

Using the notations defined above, the input-output relations describing the considered relay channel in symbol interval  $i$  are given as

$$\hat{Y}_1[i] = h_{SR}X_1[i] + h_{RR}[i]X_2[i] + \hat{N}_1[i] \quad (1)$$

$$\hat{Y}_2[i] = h_{RD}X_2[i] + \hat{N}_2[i]. \quad (2)$$

Furthermore, an average “per-node” power constraint is assumed, i.e.,

$$E\{X_\beta^2[i]\} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n X_\beta^2[k] \leq P_\beta, \quad \beta \in \{1, 2\}, \quad (3)$$

where  $E\{\cdot\}$  denotes statistical expectation, and  $P_1$  and  $P_2$  are the average power constraints at the source and the relay, respectively. In the following, we model the self-interference.

### B. Self-Interference Model

As pointed out above, the self-interference channel gain,  $h_{RR}[i]$ , varies even when fading is not present, see [6]. The variations of the self-interference channel gain,  $h_{RR}[i]$ , are due to the cumulative effects of various residual distortions originating from noise, carrier frequency offset, oscillator phase noise, analog-to-digital/digital-to-analog (AD/DA) conversion imperfections, I/Q imbalance, power amplifier nonlinearity, imperfect channel estimation, etc., see [6], [7], [31]. These residual distortions<sup>2</sup> have a significant impact on the self-interference channel gain due to the very small distance between the transmitter-end and the receiver-end of the self-interference channel. Moreover, the variations of the self-interference channel gain,  $h_{RR}[i]$ , are random and cannot be accurately estimated at the FD node [6], [7], [31]. In [6], the variations caused by the residual distortions are assumed to be constant during one frame comprised of several symbols. However, the experimental results presented in [31] revealed that the residual distortions cause symbol-by-symbol variations which can be accurately modeled as independent and identically distributed (i.i.d.) Gaussian RVs<sup>3</sup> across different symbol intervals  $i$ . As a result, in this paper, we assume that  $h_{RR}[i]$  varies randomly in a symbol-by-symbol manner and is non-zero mean i.i.d. Gaussian distributed across different symbol intervals  $i$ . Hence, using the approach in [32] and without loss of generality, we model the self-interference channel gain,  $h_{RR}[i]$ , as comprised of a deterministic and an i.i.d. random component. In particular, in symbol interval  $i$ , the self-interference channel gain is modeled as

$$h_{RR}[i] = \bar{h}_{RR} + \hat{h}_{RR}[i], \quad (4)$$

<sup>2</sup>We note that similar distortions are also present in the source-relay and relay-destination channels. However, due to the large distance between transmitter and receiver, the impact of these distortions on the channel gains  $h_{SR}$  and  $h_{RD}$  is negligible.

<sup>3</sup>Similar models for the residual distortion have been employed in [13] and [14].

where  $\bar{h}_{RR}$  and  $\hat{h}_{RR}[i] \sim \mathcal{N}(0, \hat{\alpha})$  are the deterministic and random components of the self-interference channel gain, respectively. Hence,  $\bar{h}_{RR}$  and  $\hat{\alpha}$  are the mean and the variance of the self-interference channel gain  $h_{RR}[i]$ , respectively. The variance  $\hat{\alpha}$  can also be interpreted as a self-interference amplification factor (i.e.,  $1/\hat{\alpha}$  is the self-interference suppression factor). Inserting (4) into (1), we obtain the received symbol at the relay in symbol interval  $i$  as

$$\hat{Y}_1[i] = h_{SR}X_1[i] + \bar{h}_{RR}X_2[i] + \hat{h}_{RR}[i]X_2[i] + \hat{N}_1[i]. \quad (5)$$

Given sufficient time, the relay can estimate the deterministic component of the self-interference channel gain  $\bar{h}_{RR}$ , see [32]. As a result, the relay can subtract  $\bar{h}_{RR}X_2[i]$  from the received symbol  $\hat{Y}_1[i]$  since it also knows its own transmit symbol  $X_2[i]$ . Subsequently, dividing the received symbol by  $h_{SR}$ , we obtain a new received symbol at the relay in symbol interval  $i$ , denoted by  $Y_1[i]$ , given by

$$\begin{aligned} Y_1[i] &= X_1[i] + \frac{\hat{h}_{RR}[i]}{h_{SR}}X_2[i] + \frac{\hat{N}_1[i]}{h_{SR}} \\ &= X_1[i] + S[i] + N_1[i], \end{aligned} \quad (6)$$

where

$$S[i] = \frac{\hat{h}_{RR}[i]}{h_{SR}}X_2[i] \quad (7)$$

is the residual self-interference at the relay and  $N_1[i] = \hat{N}_1[i]/h_{SR}$  is the normalized noise at the relay distributed according to  $N_1[i] \sim \mathcal{N}(0, \sigma_1^2)$ , where  $\sigma_1^2 = \hat{\sigma}_1^2/h_{SR}^2$ . The residual self-interference,  $S[i]$ , is dependent on the transmit symbol at the relay,  $X_2[i]$ , and, conditioned on  $X_2[i]$ , it has the same type of distribution as the random component of the self-interference channel gain,  $\hat{h}_{RR}[i]$ , i.e., an i.i.d. Gaussian distribution. Let  $\alpha$  be defined as  $\alpha = \hat{\alpha}/h_{SR}^2$ , which can be interpreted as the normalized self-interference amplification factor. Using  $\alpha$  and assuming that the transmit symbol at the relay in symbol interval  $i$  is  $X_2[i] = x_2[i]$ , the distribution of the residual self-interference,  $S[i]$ , can be written as

$$S[i] \sim \mathcal{N}(0, \alpha x_2^2[i]), \quad \text{if } X_2[i] = x_2[i]. \quad (8)$$

*Remark 1:* We note that modeling the self-interference channel gain,  $h_{RR}[i]$ , as an i.i.d. Gaussian RV leads to the worst-case scenario in terms of capacity for the considered relay channel. In particular, let us assume that  $\hat{h}_{RR}[i]$  has a constrained second moment, which is a reasonable assumption considering that in reality the self-interference cannot have infinite power. Then, since  $X_2[i]$  also has a constrained second moment, c.f. (3), the worst-case scenario in terms of capacity is if we assume that  $\hat{h}_{RR}[i]$  is a zero-mean i.i.d. Gaussian RV, since a Gaussian RV has the highest

uncertainty (i.e., entropy) among all possible RVs with constrained second moment [29]. Hence, modeling the self-interference channel gain,  $h_{RR}[i]$ , as an i.i.d. Gaussian RV is not only motivated by the experimental results in [31], but also leads to a lower bound on the capacity for other distributions of  $h_{RR}[i]$ .

To obtain also a normalized received symbol at the destination, we normalize  $\hat{Y}_2[i]$  in (2) by  $h_{RD}$ , which yields

$$Y_2[i] = X_2[i] + N_2[i]. \quad (9)$$

In (9),  $N_2[i]$  is the normalized noise power at the destination distributed as  $N_2[i] = \mathcal{N}(0, \sigma_2^2)$ , where  $\sigma_2^2 = \hat{\sigma}_2^2/h_{RD}^2$ .

Now, instead of deriving the capacity of the considered relay channel using the input-output relations in (1) and (2), we can derive the capacity using an equivalent relay channel defined by the input-output relations in (6) and (9), respectively, where, in symbol interval  $i$ ,  $X_1[i]$  and  $X_2[i]$  are the inputs at source and relay, respectively,  $Y_1[i]$  and  $Y_2[i]$  are the outputs at relay and destination, respectively,  $N_1[i]$  and  $N_2[i]$  are AWGNs with variances  $\sigma_1^2 = \hat{\sigma}_1^2/h_{SR}^2$  and  $\sigma_2^2 = \hat{\sigma}_2^2/h_{RD}^2$ , respectively, and  $S[i]$  is the residual self-interference with conditional distribution given by (8), which is a function of the normalized self-interference amplification factor  $\alpha$ .

Having defined the system model, in the following, we derive the corresponding channel capacity.

### III. CAPACITY

In this section, we study the capacity of the considered Gaussian two-hop FD relay channel with self-interference.

#### A. Derivation of the Capacity

To derive the capacity of the considered relay channel, we first assume that RVs  $X_1$  and  $X_2$ , which model the transmit symbols at source and relay, take values  $x_1$  and  $x_2$  from sets  $\mathcal{X}_1$  and  $\mathcal{X}_2$ , respectively. Now, since the considered relay channel belongs to the class of memoryless degraded relay channels defined in [2], its capacity is given by [2, Theorem 1]

$$\begin{aligned} C &= \max_{p(x_1, x_2) \in \mathcal{P}} \min \{I(X_1; Y_1 | X_2), I(X_2; Y_2)\} \\ \text{Subject to C1: } &E\{X_1^2\} \leq P_1 \\ \text{C2: } &E\{X_2^2\} \leq P_2, \end{aligned} \quad (10)$$

where  $\mathcal{P}$  is a set which contains all valid distributions. In order to obtain the capacity in (10), we need to find the optimal joint input distribution,  $p(x_1, x_2)$ , which maximizes the  $\min\{\cdot\}$  function in

(10) and satisfies constraints C1 and C2. To this end, note that  $p(x_1, x_2)$  can be written equivalently as  $p(x_1, x_2) = p(x_1|x_2)p(x_2)$ . Using this relation, we can represent the maximization in (10) equivalently as two nested maximizations, one with respect to  $p(x_1|x_2)$  for a fixed  $p(x_2)$ , and the other one with respect to  $p(x_2)$ . Thereby, we can write the capacity in (10) equivalently as

$$C = \max_{p(x_2) \in \mathcal{P}} \max_{p(x_1|x_2) \in \mathcal{P}} \min\{I(X_1; Y_1|X_2), I(X_2; Y_2)\}$$

Subject to C1:  $E\{X_1^2\} \leq P_1$

C2:  $E\{X_2^2\} \leq P_2.$  (11)

Now, since in the  $\min\{\cdot\}$  function in (11) only  $I(X_1; Y_1|X_2)$  is dependent on the distribution  $p(x_1|x_2)$ , whereas  $I(X_2; Y_2)$  does not depend on  $p(x_1|x_2)$ , we can write the capacity expression in (11) equivalently as

$$C = \max_{p(x_2) \in \mathcal{P}} \min \left\{ \max_{p(x_1|x_2) \in \mathcal{P}} I(X_1; Y_1|X_2), I(X_2; Y_2) \right\}$$

Subject to C1:  $E\{X_1^2\} \leq P_1$

C2:  $E\{X_2^2\} \leq P_2.$  (12)

Hence, to obtain the capacity of the considered relay channel, we first need to find the conditional input distribution at the source,  $p(x_1|x_2)$ , which maximizes  $I(X_1; Y_1|X_2)$  such that constraint C1 holds. Next, we need to find the optimal input distribution at the relay,  $p(x_2)$ , which maximizes the  $\min\{\cdot\}$  expression in (12) such that constraints C1 and C2 hold. In the following, we first derive the optimal input distribution at the source  $p(x_1|x_2)$ .

### B. Optimal Input Distribution at the Source $p^*(x_1|x_2)$

The optimal input distribution at the source which achieves the capacity in (12), denoted by  $p^*(x_1|x_2)$ , is given in the following theorem.

*Theorem 1:* The optimal input distribution at the source  $p^*(x_1|x_2)$ , which achieves the capacity of the considered relay channel in (12), is the zero-mean Gaussian distribution with variance  $P_1(x_2)$  given by

$$P_1(x_2) = \alpha \max\{0, x_{\text{th}}^2 - x_2^2\}, \quad (13)$$

where  $x_{\text{th}}$  is a positive threshold constant found as follows. If  $p(x_2)$  is a discrete distribution,  $x_{\text{th}}$  is found as the solution of the following identity

$$\sum_{x_2 \in \mathcal{X}_2} \alpha \max\{0, x_{\text{th}}^2 - x_2^2\} p(x_2) = P_1, \quad (14)$$



and the corresponding  $\max_{p(x_1|x_2) \in \mathcal{P}} I(X_1; Y_1|X_2)$  is obtained as

$$\max_{p(x_1|x_2) \in \mathcal{P}} I(X_1; Y_1|X_2) = \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2). \quad (15)$$

Otherwise, if  $p(x_2)$  is a continuous distribution, the sums in (14) and (15) have to be replaced by integrals.

*Proof:* Please refer to Appendix A. ■

From Theorem 1, we can see that the source should perform power allocation in a symbol-by-symbol manner. In particular, the average power of the source's transmit symbols,  $P_1(x_2)$ , given by (13), depends on the amplitude of the transmit symbol at the relay,  $|x_2|$ . The lower the amplitude of the transmit symbol of the relay is, the higher the average power of the source's transmit symbols should be since, in that case, there is a high probability for weak self-interference. Conversely, the higher the amplitude of the transmit symbol of the relay is, the lower the average power of the source's transmit symbols should be since, in that case, there is a high probability for strong self-interference. If the amplitude of the transmit symbol of the relay is larger than the threshold  $x_{\text{th}}$ , the chance for very strong self-interference becomes too high, and the source remains silent to conserve energy for the cases when the self-interference is weaker.

In the following, we derive the optimal input distribution at the relay.

### C. Optimal Input Distribution at the Relay $p^*(x_2)$

The optimal input distribution at the relay, denoted by  $p^*(x_2)$ , which achieves the capacity of the considered relay channel is given in the following theorem.

*Theorem 2:* If condition

$$\log_2 \left( 1 + \frac{P_2}{\sigma_2^2} \right) \leq \int_{-x_{\text{th}}}^{x_{\text{th}}} \log_2 \left( 1 + \frac{\alpha(x_{\text{th}}^2 - x_2^2)}{\sigma_1^2 + \alpha x_2^2} \right) \frac{1}{\sqrt{2\pi P_2}} e^{-\frac{x_2^2}{2P_2}} dx_2 \quad (16)$$

holds, where the amplitude threshold  $x_{\text{th}}$  is found from

$$\sqrt{\frac{2P_2}{\pi}} \alpha x_{\text{th}} \exp \left( -\frac{x_{\text{th}}^2}{2P_2} \right) + \alpha(x_{\text{th}}^2 - P_2) \text{erf} \left( \frac{x_{\text{th}}}{\sqrt{2P_2}} \right) = P_1, \quad (17)$$

with  $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ , then the optimal input distribution at the relay,  $p^*(x_2)$ , is the zero-mean Gaussian distribution with variance  $P_2$  and the corresponding capacity of the considered relay channel is given by

$$C = \frac{1}{2} \log_2 \left( 1 + \frac{P_2}{\sigma_2^2} \right). \quad (18)$$

Otherwise, if condition (16) does not hold, then the optimal input distribution at the relay,  $p^*(x_2)$  is discrete and symmetric with respect to  $x_2 = 0$ . Furthermore, the capacity and the optimal



input distribution at the relay,  $p^*(x_2)$ , can be found by solving the following concave optimization problem

$$\begin{aligned}
C &= \max_{p(x_2) \in \mathcal{P}} \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2) \\
\text{Subject to C1: } &\sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2) \leq I(X_2; Y_2) \\
\text{C2: } &\sum_{x_2 \in \mathcal{X}_2} x_2^2 p(x_2) \leq P_2 \\
\text{C3: } &\sum_{x_2 \in \mathcal{X}_2} \alpha \max\{0, x_{\text{th}}^2 - x_2^2\} p(x_2) = P_1.
\end{aligned} \tag{19}$$

Moreover, solving (19) reveals that constraint C1 has to hold with equality and that  $p^*(x_2)$  has the following discrete form

$$p^*(x_2) = p_{2,0} \delta(x_2) + \sum_{j=1}^J \frac{1}{2} p_{2,j} (\delta(x_2 - x_{2,j}) + \delta(x_2 + x_{2,j})), \tag{20}$$

where  $p_{2,j} \in [0, 1]$  is the probability that  $X_2 = x_{2,j}$  will occur, where  $x_{2,j} > 0$  and  $\sum_{j=0}^J p_{2,j} = 1$  hold. With  $p^*(x_2)$  as in (20), the capacity has the following general form

$$C = \frac{p_{2,0}}{2} \log_2 \left( 1 + \frac{\alpha x_{\text{th}}^2}{\sigma_1^2} \right) + \sum_{j=1}^J \frac{p_{2,j}}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_{2,j}^2\}}{\sigma_1^2 + \alpha x_{2,j}^2} \right). \tag{21}$$

*Proof:* Please refer to Appendix B. ■

From Theorem 2, we can draw the following conclusions. If condition (16) holds, then the relay-destination channel is the bottleneck link. In particular, even if the relay transmits with a zero-mean Gaussian distribution with which it achieves the capacity of the relay-destination channel, the capacity of the relay-destination channel is still smaller than the mutual information (i.e., data rate) of the source-relay channel. Otherwise, if condition (16) does not hold, then the optimal input distribution at the relay,  $p^*(x_2)$ , is always discrete and symmetric with respect to  $x_2 = 0$ . Moreover, in this case, the mutual informations of the source-relay and relay-destination channels have to be equal, i.e.,  $I(X_1; Y_1 | X_2) \big|_{p(x_2)=p^*(x_2)} = I(X_2; Y_2) \big|_{p(x_2)=p^*(x_2)}$  has to hold. In addition, we note that constraint C2 in (19) does not always have to hold with equality, i.e., in certain cases it is optimal for the relay to reduce its average power. In particular, if the relay-destination channel is very strong compared to the source-relay channel, then, by reducing the average power of the relay, we reduce the average power of the self-interference on the source-relay channel, and thereby improve the quality of the source-relay channel. Note that, even if the average power of the relay is reduced,  $I(X_1; Y_1 | X_2) \big|_{p(x_2)=p^*(x_2)} = I(X_2; Y_2) \big|_{p(x_2)=p^*(x_2)}$  still has to hold.

*Remark 2:* From (17), it can be observed that threshold  $x_{\text{th}}$  is inversely proportional to the normalized self-interference amplification factor  $\alpha$ . In other words, the smaller  $\alpha$  is, the larger  $x_{\text{th}}$  becomes. In the limit, when  $\alpha \rightarrow 0$ , we have  $x_{\text{th}} \rightarrow \infty$ . This is expected since for smaller  $\alpha$ , the average power of the self-interference also becomes smaller, which allows the source to transmit more frequently. If  $\alpha \rightarrow 0$  the self-interference tends to zero. Consequently, the source should never be silent, i.e.,  $x_{\text{th}} \rightarrow \infty$ , which is in line with the optimal behavior of the source for the case of ideal FD relaying without self-interference described in [2]. On the other hand, inserting the solution for  $x_{\text{th}}$  from (17) into (16), and then evaluating (16), it can be observed that the right hand-side of (16) is a strictly decreasing function of  $\alpha$ . This is expected since larger  $\alpha$  result in a larger average self-interference power and thereby in a smaller achievable rate on the source-relay channel.

#### D. Achievability of the Capacity

The source wants to transmit message  $W$  to the destination, which is drawn uniformly from a message set  $\{1, 2, \dots, 2^{nR}\}$  and carries  $nR$  bits of information. To this end, the transmission time is split into  $B + 1$  time slots, where each time slot is comprised of  $k$  symbol intervals. Moreover, message  $W$  is split into  $B$  messages, denoted by  $w(1), \dots, w(B)$ , where each  $w(b)$ , for  $b = 1, \dots, B$ , carries  $kR$  bits of information. Each of these messages is to be transmitted in a different time slot. In particular, in time slot one, the source sends message  $w(1)$  during  $k$  symbol intervals to the relay and the relay is silent. In time slot  $b$ , for  $b = 2, \dots, B$ , source and relay send messages  $w(b)$  and  $w(b - 1)$  to relay and destination during  $k$  symbol intervals, respectively. In time slot  $B + 1$ , the relay sends message  $w(B)$  to the destination during  $k$  symbol intervals and the source is silent. Hence, in the first time slot, the relay is silent since it does not have information to transmit, and in time slot  $B + 1$ , the source is silent since it has no more information to transmit. In time slots 2 to  $B$ , both source and relay transmit. During the  $B + 1$  time slots, the channel is used  $k(B + 1)$  times to send  $nR = BkR$  bits of information, leading to an overall information rate of

$$\lim_{B \rightarrow \infty} \lim_{k \rightarrow \infty} \frac{BkR}{k(B + 1)} = R \text{ bits/symbol.} \quad (22)$$

A detailed description of the proposed coding scheme for each time slot is given in the following, where we explain the rates, codebooks, encoding, and decoding used for transmission.

*Rates:* The transmission rate of both source and relay is denoted by  $R$  and given by

$$R = C - \epsilon, \quad (23)$$

where  $C$  is given in Theorem 2 and  $\epsilon > 0$  is an arbitrarily small number.

*Codebooks:* We have two codebooks, namely, the source's transmission codebook and the relay's transmission codebook. The source's transmission codebook is generated by mapping each possible binary sequence comprised of  $kR$  bits, where  $R$  is given by (23), to a codeword  $\mathbf{x}_1$  comprised of  $kp_T$  symbols, where  $p_T$  is the following probability

$$p_T = \Pr \{ |x_2| < x_{\text{th}} \}. \quad (24)$$

Hence,  $p_T$  is the probability that the relay will transmit a symbol with an amplitude which is smaller than the threshold  $x_{\text{th}}$ . In other words,  $p_T$  is the fraction of symbols in the relay's codeword which have an amplitude which is smaller than the threshold  $x_{\text{th}}$ . The symbols in each codeword  $\mathbf{x}_1$  are generated independently according to the zero-mean *unit variance* Gaussian distribution. Since in total there are  $2^{kR}$  possible binary sequences comprised of  $kR$  bits, with this mapping, we generate  $2^{kR}$  codewords  $\mathbf{x}_1$  each comprised of  $kp_T$  symbols. These  $2^{kR}$  codewords form the source's transmission codebook, which we denote by  $\mathcal{C}_1$ .

On the other hand, the relay's transmission codebook is generated by mapping each possible binary sequence comprised of  $kR$  bits, where  $R$  is given by (23), to a transmission codeword  $\mathbf{x}_2$  comprised of  $k$  symbols. Note that the length of the relay's codewords,  $\mathbf{x}_2$ , is larger than the length of the source's codewords,  $\mathbf{x}_1$ , i.e.,  $k > kp_T$ . The symbols in each codeword  $\mathbf{x}_2$  are generated independently according to the optimal distribution  $p^*(x_2)$  given in Theorem 2. The  $2^{kR}$  codewords  $\mathbf{x}_2$  form the relay's transmission codebook denoted by  $\mathcal{C}_2$ .

The two codebooks are known at all three nodes. Moreover, the power allocation policy at the source,  $P_1(x_2)$ , given in (13), is assumed to be known at source and relay.

*Encoding, Transmission, and Decoding:* In the first time slot, the source maps  $w(1)$  to the appropriate codeword  $\mathbf{x}_1(1)$  from its codebook  $\mathcal{C}_1$ . Then, codeword  $\mathbf{x}_1(1)$  is transmitted to the relay, where each symbol of  $\mathbf{x}_1(1)$  is amplified by  $\sqrt{P_1(x_2 = 0)}$ , where  $P_1(x_2)$  is given in (13). On the other hand, the relay is scheduled to always receive and be silent (i.e., to set its transmit symbol to zero) during the first time slot. However, knowing that the codeword transmitted by the source  $\mathbf{x}_1(1)$  is comprised of  $kp_T$  symbols, the relay constructs the received codeword, denoted by  $\mathbf{y}_1(1)$ , only from the first  $kp_T$  received symbols.

*Lemma 1:* The codeword  $\mathbf{x}_1(1)$  sent in the first time slot can be decoded successfully from the codeword received at the relay,  $\mathbf{y}_1(1)$ , using a typical decoder [29] since  $R$  satisfies

$$R < \max_{p(x_1|x_2=0)} I(X_1; Y_1 | X_2 = 0) p_T = \frac{1}{2} \log_2 \left( 1 + \frac{\alpha x_{\text{th}}^2}{\sigma_1^2} \right) p_T. \quad (25)$$

*Proof:* Please refer to Appendix D. ■

In time slots  $b = 2, \dots, B$ , the encoding, transmission, and decoding are performed as follows. In time slots  $b = 2, \dots, B$ , the source and the relay map  $w(b)$  and  $w(b-1)$  to the appropriate codewords

$\mathbf{x}_1(b)$  and  $\mathbf{x}_2(b)$  from codebooks  $\mathcal{C}_1$  and  $\mathcal{C}_2$ , respectively. Note that the source also knows  $\mathbf{x}_2(b)$  since  $\mathbf{x}_2(b)$  was generated from  $w(b-1)$  which the source transmitted in the previous (i.e., the  $(b-1)$ -th) time slot. As a result, both source and relay know the symbols in  $\mathbf{x}_2(b)$  and can determine whether their amplitudes are smaller or larger than the threshold  $x_{\text{th}}$ . Hence, if the amplitude of the first symbol in codeword  $\mathbf{x}_2(b)$  is smaller than the threshold  $x_{\text{th}}$ , then, in the first symbol interval of time slot  $b$ , the source transmits the first symbol from codeword  $\mathbf{x}_1(b)$  amplified by  $\sqrt{P_1(x_{2,1})}$ , where  $x_{2,1}$  is the first symbol in relay's codeword  $\mathbf{x}_2(b)$  and  $P_1(x_2)$  is given by (13). Otherwise, if the amplitude of the first symbol in codeword  $\mathbf{x}_2(b)$  is larger than the threshold  $x_{\text{th}}$ , then the source is silent. The same procedure is performed for the  $j$ -th symbol interval in time slot  $b$ , for  $j = 1, \dots, k$ . In particular, if the amplitude of the  $j$ -th symbol in codeword  $\mathbf{x}_2(b)$  is smaller than threshold  $x_{\text{th}}$ , then in the  $j$ -th symbol interval of time slot  $b$ , the source transmits its next untransmitted symbol from codeword  $\mathbf{x}_1(b)$  amplified by  $\sqrt{P_1(x_{2,j})}$ , where  $x_{2,j}$  is the  $j$ -th symbol in relay's codeword  $\mathbf{x}_2(b)$ . Otherwise, if the amplitude of the  $j$ -th symbol in codeword  $\mathbf{x}_2(b)$  is larger than threshold  $x_{\text{th}}$ , then for the  $j$ -th symbol interval of time slot  $b$ , the source is silent. On the other hand, the relay transmits all symbols from  $\mathbf{x}_2(b)$  while simultaneously receiving. Let  $\hat{\mathbf{y}}_1(b)$  denote the received codeword at the relay in time slot  $b$ . Then, the relay discards those symbols from the received codeword,  $\hat{\mathbf{y}}_1(b)$ , for which the corresponding symbols in  $\mathbf{x}_2(b)$  have amplitudes which exceed the threshold  $x_{\text{th}}$ , and only collects the symbols in  $\hat{\mathbf{y}}_1(b)$  for which the corresponding symbols in  $\mathbf{x}_2(b)$  have amplitudes which are smaller than  $x_{\text{th}}$ . The symbols collected from  $\hat{\mathbf{y}}_1(b)$  constitute the relay's information-carrying received codeword, denoted by  $\mathbf{y}_1(b)$ , which is used for decoding.

*Lemma 2:* The codewords  $\mathbf{x}_1(b)$  sent in time slots  $b = 2, \dots, B$  can be decoded successfully at the relay from the corresponding received codewords  $\mathbf{y}_1(b)$ , respectively, using a jointly typical decoder since  $R$  satisfies

$$R < \sum_{x_2 \in \mathcal{X}_2} \max_{p(x_1|x_2)} I(X_1; Y_1 | X_2 = x_2) p^*(x_2) = \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p^*(x_2). \quad (26)$$

*Proof:* Please refer to Appendix E. ■

On the other hand, the destination listens during the entire time slot  $b$  and receives a codeword denoted by  $\mathbf{y}_2(b)$ . By following the “standard” method for analyzing the probability of error for rates smaller than the capacity, given in [29, Sec. 7.7], it can be shown in a straightforward manner that the destination can successfully decode  $\mathbf{x}_2(b)$  from the received codeword  $\mathbf{y}_2(b)$ , and thereby obtain  $w(b-1)$ , since rate  $R$  satisfies

$$R < I(X_2; Y_2) \Big|_{p(x_2)=p^*(x_2)}, \quad (27)$$

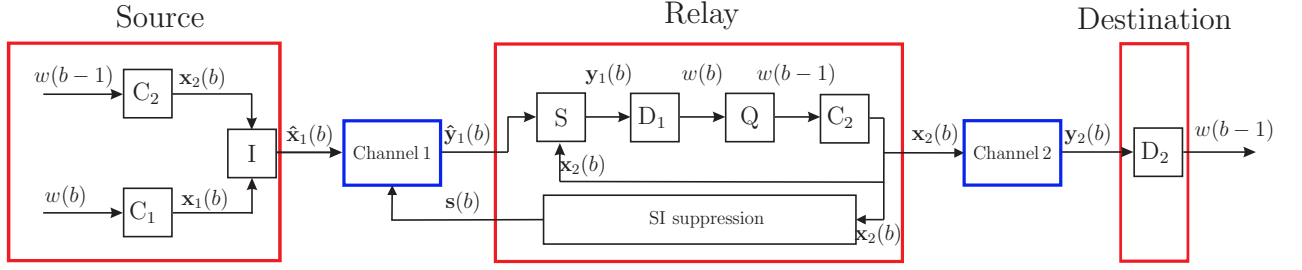


Fig. 1. Block diagram of the proposed channel coding protocol for time slot  $b$ . The following notations are used in the block diagram:  $C_1$  and  $C_2$  are encoders,  $D_1$  and  $D_2$  are decoders,  $I$  is an inserter,  $S$  is a selector,  $Q$  is a buffer,  $s(b)$  is the residual self-interference (SI) vector in time slot  $b$ , and  $w(b)$  denotes the message transmitted by the source in time slot  $b$ .

where  $I(X_2; Y_2)$  is given in Theorem 2.

In the last (i.e., the  $(B + 1)$ -th) time slot, the source is silent and the relay transmits  $w(B)$  by mapping it to the corresponding codeword  $\mathbf{x}_2(B + 1)$  from codebook  $\mathcal{C}_2$ . The relay transmits all symbols in codeword  $\mathbf{x}_2(B + 1)$  to the destination during time slot  $B + 1$ . The destination can decode the received codeword in time slot  $B + 1$  successfully, since (27) holds.

Finally, since both relay and destination can decode their respective codewords in each time slot, the entire message  $W$  can be decoded successfully at the destination at the end of the  $(B + 1)$ -th time slot.

A block diagram of the proposed coding scheme is presented in Fig. 1. In particular, in Fig. 1, we show schematically the encoding, transmission, and decoding at source, relay, and destination. The flow of encoding/decoding in Fig. 1 is as follows. Messages  $w(b - 1)$  and  $w(b)$  are encoded into  $\mathbf{x}_2(b)$  and  $\mathbf{x}_1(b)$  at the source using the encoders  $C_2$  and  $C_1$ , respectively. Then, an inserter  $I$  is used to create a vector  $\hat{\mathbf{x}}_1(b)$  by inserting the symbols of  $\mathbf{x}_1(b)$  into the positions of  $\hat{\mathbf{x}}_1(b)$  for which the corresponding elements of  $\mathbf{x}_2(b)$  have amplitudes smaller than  $x_{\text{th}}$  and setting all other symbols in  $\hat{\mathbf{x}}_1(b)$  to zero. Hence, vector  $\hat{\mathbf{x}}_1(b)$  is identical to codeword  $\mathbf{x}_1(b)$  except for the added silent (i.e., zero) symbols generated at the source. The source then transmits  $\hat{\mathbf{x}}_1(b)$  and the relay receives the corresponding codeword  $\hat{\mathbf{y}}_1(b)$ . Simultaneously, the relay encodes  $w(b - 1)$  into  $\mathbf{x}_2(b)$  using encoder  $C_2$  and transmits it to the destination, which receives codeword  $\mathbf{y}_2(b)$ . Next, using  $\mathbf{x}_2(b)$ , the relay constructs  $\mathbf{y}_1(b)$  from  $\hat{\mathbf{y}}_1(b)$  by selecting only those symbols from  $\hat{\mathbf{y}}_1(b)$  for which the corresponding symbols in  $\mathbf{x}_2(b)$  have amplitudes smaller than  $x_{\text{th}}$ . Using decoder  $D_1$ , the relay then decodes  $\mathbf{y}_1(b)$  into  $w(b)$  and stores the decoded bits in its buffer  $Q$ . On the other hand, the destination decodes  $\mathbf{y}_2(b)$  into  $w(b - 1)$  using decoder  $D_2$ .

### E. Analytical Expression for Tight Lower Bound on the Capacity

For the non-trivial case when the relay-destination link is not the bottleneck, the capacity of the Gaussian two-hop FD relay channel with self-interference is given in the form of an optimization problem, cf. (19), which is not suitable for analysis. As a result, in this subsection, we propose a suboptimal input distribution at the relay, which yields an analytical expression for a lower bound on the capacity. Our numerical results show that this lower bound is tight, at least for the considered numerical examples cf. Fig. 3. In particular, we propose the relay to use the following input distribution

$$p(x_2) = p_B(x_2) = q \frac{1}{\sqrt{2\pi P_2/q}} \exp\left(-\frac{x_2^2}{2P_2/q}\right) + (1-q)\delta(x_2). \quad (28)$$

Hence, with probability  $q$ , the relay transmits a symbol from a zero-mean Gaussian distribution with variance  $P_2/q$ , and is silent with probability  $1-q$ . Since the relay transmits only in  $q$  fraction of the time, the average transmit power when the relay transmits is set to  $P_2/q$  in order for the average transmit power during the entire transmission time to be  $P_2$ . Now, with the input distribution  $p_B(x_2)$  in (28), we obtain the mutual information of the source-relay channel as

$$\begin{aligned} & \max_{p(x_1|x_2) \in \mathcal{P}} I(X_1; Y_1 | X_2) \Big|_{p(x_2)=p_B(x_2)} \\ &= q \int_{-x_{\text{th}}}^{x_{\text{th}}} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha(x_{\text{th}}^2 - x_2^2)}{\sigma_1^2 + \alpha x_2^2} \right) \frac{1}{\sqrt{2\pi P_2/q}} \exp\left(-\frac{x_2^2}{2P_2/q}\right) dx_2 + (1-q) \frac{1}{2} \log_2 \left( 1 + \frac{\alpha x_{\text{th}}^2}{\sigma_1^2} \right), \end{aligned} \quad (29)$$

and the mutual information of the relay-destination channel as

$$\begin{aligned} & I(X_2; Y_2) \Big|_{p(x_2)=p_B(x_2)} \\ &= - \int_{-\infty}^{\infty} \left[ q \frac{1}{\sqrt{2\pi(P_2/q + \sigma_2^2)}} \exp\left(-\frac{y_2^2}{2(P_2/q + \sigma_2^2)}\right) + (1-q) \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{y_2^2}{2\sigma_2^2}\right) \right] \\ & \quad \times \log_2 \left( q \frac{1}{\sqrt{2\pi(P_2/q + \sigma_2^2)}} \exp\left(-\frac{y_2^2}{2(P_2/q + \sigma_2^2)}\right) + (1-q) \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{y_2^2}{2\sigma_2^2}\right) \right) \\ & \quad - \frac{1}{2} \log_2(2\pi e \sigma_2^2). \end{aligned} \quad (30)$$

The threshold  $x_{\text{th}}$  in (29) and the probability  $q$  in (29) and (30) are found from the following system of two equations

$$\begin{cases} q \left( \sqrt{\frac{2P_2/q}{\pi}} \alpha x_{\text{th}} \exp\left(-\frac{x_{\text{th}}^2}{2P_2/q}\right) + \alpha(x_{\text{th}}^2 - P_2/q) \text{erf}\left(\frac{x_{\text{th}}}{\sqrt{2P_2/q}}\right) \right) + (1-q)\alpha x_{\text{th}}^2 = P_1 \\ \max_{p(x_1|x_2) \in \mathcal{P}} I(X_1; Y_1 | X_2) \Big|_{p(x_2)=p_B(x_2)} = I(X_2; Y_2) \Big|_{p(x_2)=p_B(x_2)} \end{cases}. \quad (31)$$

The achievable rate with the suboptimal input distribution  $p_B(x_2)$  is found by inserting  $x_{\text{th}}$  and  $q$  found from (31) into (29) and/or (30).

#### IV. NUMERICAL EVALUATION

In this section, we numerically evaluate the capacity of the considered two-hop FD relay channel with self-interference and compare it to several benchmark schemes. To this end, we first provide the system parameters, introduce benchmark schemes, and then present the numerical results.

##### A. System Parameters

We compute the channel gains of the source-relay ( $SR$ ) and relay-destination ( $RD$ ) links using the standard path loss model

$$h_L^2 = \left( \frac{c}{f_c 4\pi} \right)^2 d_L^{-\gamma}, \quad \text{for } L \in \{SR, RD\}, \quad (32)$$

where  $c$  is the speed of light,  $f_c$  is the carrier frequency,  $d_L$  is the distance between the transmitter and the receiver of link  $L$ , and  $\gamma$  is the path loss exponent. For the numerical examples in this section, we assume  $\gamma = 3$ ,  $d_{SR} = 500$  meter, and  $d_{RD} = 500$  meter or  $d_{RD} = 300$  meter. Moreover, we assume a carrier frequency of  $f_c = 2.4$  GHz. The transmit bandwidth is assumed to be 200 kHz. Furthermore, we assume that the noise power per Hz is  $-170$  dBm, which for 200 kHz leads to a total noise power of  $2 \times 10^{-15}$  Watt. Finally, the normalized self-interference amplification factor,  $\alpha$ , is computed as  $\alpha = \hat{\alpha}/h_{SR}^2$ , where  $\hat{\alpha}$  is the self-interference amplification factor. For our numerical results, we will assume that the self-interference amplification factor  $\hat{\alpha}$  ranges from  $-110$  dB to  $-140$  dB, hence, the self-interference suppression factor,  $1/\hat{\alpha}$ , ranges from 110 dB to 140 dB.

##### B. Benchmark Schemes

*Benchmark Scheme 1 (Ideal FD Transmission without Self-Interference):* The idealized case is when the relay can cancel all of its self-interference. For this case, the capacity of the Gaussian two-hop FD relay channel without self-interference is given in [2] as

$$C_{\text{FD,Ideal}} = \min \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{P_1}{\sigma_1^2} \right), \frac{1}{2} \log_2 \left( 1 + \frac{P_2}{\sigma_2^2} \right) \right\}. \quad (33)$$

The optimal input distributions at source and relay are zero-mean Gaussian with variances  $P_1$  and  $P_2$ , respectively.

*Benchmark Scheme 2 (Conventional FD Transmission with Self-Interference):* The conventional FD relaying scheme for the case when the relay suffers from self-interference uses the same input



distributions at source and relay as in the ideal case when the relay does not have self-interference, i.e., the input distributions at source and relay are zero-mean Gaussian with variances  $P_1$  and  $P_2$ , respectively. In this case, the achieved rate is given by

$$R_{\text{FD,Conv}} = \min \left\{ \int_{-\infty}^{\infty} \frac{1}{2} \log_2 \left( 1 + \frac{P_1}{\sigma_1^2 + \alpha x_2^2} \right) \frac{e^{-x_2^2/(2P_2)}}{\sqrt{2\pi P_2}} dx_2 ; \frac{1}{2} \log_2 \left( 1 + \frac{P_2}{\sigma_2^2} \right) \right\}. \quad (34)$$

*Benchmark Scheme 3 (Optimal HD Transmission):* The capacity of the Gaussian two-hop HD relay channel was derived in [27], but can also be directly obtained from Theorem 2 by letting  $\alpha \rightarrow \infty$ . This capacity can be obtained numerically and will be denoted by  $C_{\text{HD}}$ . In this case, the optimal input distribution at the relay is discrete. On the other hand, the source transmits using a Gaussian input distribution with constant variance. Moreover, the source transmits only when the relay is silent, i.e., only when the relay transmits the symbol zero, otherwise, the source is silent. Since both source and relay are silent in fractions of the time, the average powers at source and relay for HD relaying are adjusted such that they are equal to the average powers at source and relay for FD relaying, respectively.

*Benchmark Scheme 4 (Conventional HD Transmission):* The conventional HD relaying scheme uses the zero-mean Gaussian distribution with variances  $P_1$  and  $P_2$  at the source and the relay, respectively. However, compared to the optimal HD transmission in [27], in conventional HD transmission, the relay alternates between receiving and transmitting in a codeword-by-codeword manner. As a result, the achieved rate is given by [28]

$$R_{\text{HD,Conv}} = \max_t \min \left\{ \frac{1-t}{2} \log_2 \left( 1 + \frac{P_1/(1-t)}{\sigma_1^2} \right) ; \frac{t}{2} \log_2 \left( 1 + \frac{P_2/t}{\sigma_2^2} \right) \right\}. \quad (35)$$

In (35), since source and relay transmit only in  $(1-t)$  and  $t$  fraction of the time, the average powers at source and relay are adjusted such that they are equal to the average powers at source and relay for FD relaying, respectively.

### C. Numerical Results

In this subsection, we denote the capacity of the considered FD relay channel, obtained from Theorem 2, by  $C_{\text{FD}}$ .

In Fig. 2, we plot the optimal input distribution at the relay,  $p^*(x_2)$ , for  $d_{SR} = d_{RD} = 500$  meter,  $P_1 = P_2 = 25$  dBm and a self-interference suppression factor of  $1/\hat{\alpha} = 130$  dB. As can be seen from Fig. 2, the relay is silent in 40% of the time, and the source transmits only when  $|x_2| < x_{\text{th}} = 0.9312$ . Hence, similar to optimal HD relaying in [27], shutting down the transmitter at the relay in a symbol-by-symbol manner plays an important role in achieving the capacity. However, in contrast to optimal HD relaying where the source transmits only when the relay is

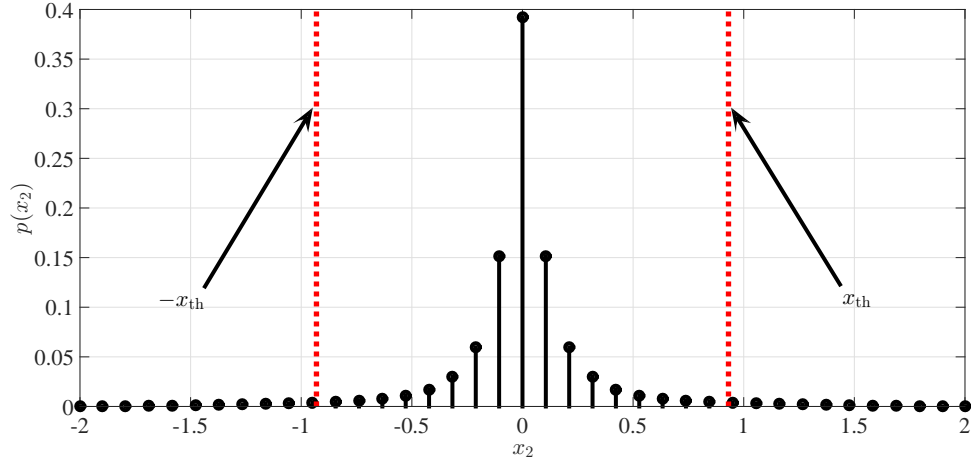


Fig. 2. Optimal input distribution at the relay,  $p^*(x_2)$ , for  $d_{SR} = d_{RD} = 500$  meter,  $P_1 = P_2 = 25$  dBm, and self-interference suppression factor,  $1/\hat{\alpha} = 130$  dB.

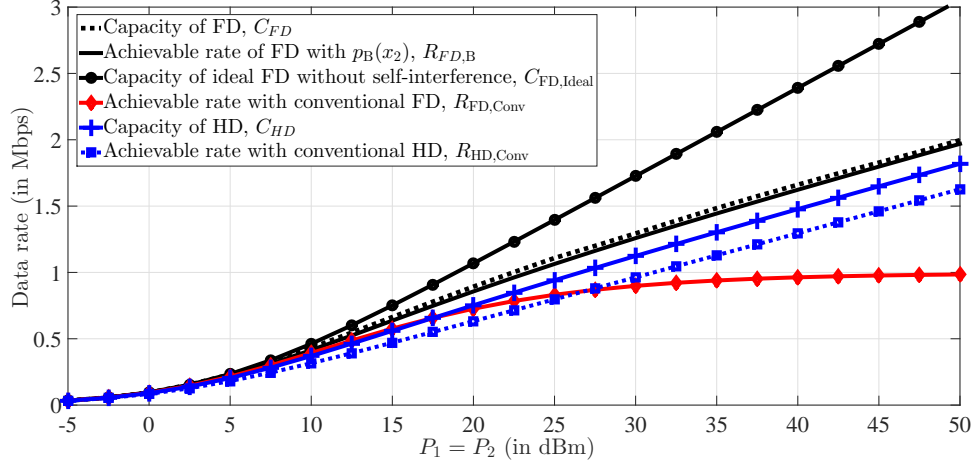


Fig. 3. Comparison of the derived capacity with the rates of the benchmark schemes as a function of the source and relay transmit powers  $P_1 = P_2$  in dBm for a self-interference suppression factor,  $1/\hat{\alpha} = 130$  dB.

silent, i.e., only when  $x_2 = 0$  occurs, in FD relaying, the source has more opportunities to transmit since it can transmit also when the relay transmits a symbol whose amplitude is smaller than  $x_{th}$ , i.e., when  $-x_{th} \leq x_2 \leq x_{th}$  holds. For the example in Fig. 2, the source transmits 96 % of the time.

In Fig. 3, we compare the capacity of the considered FD relay channel,  $C_{FD}$ , with the achievable rate with the suboptimal input distribution,  $p_B(x_2)$ , given in Section III-E, denoted by  $R_{FD,B}$ , the capacity achieved with ideal FD relaying without self-interference,  $C_{FD,Ideal}$ , cf. Benchmark Scheme 1, the rate achieved with conventional FD relaying,  $R_{FD,Conv}$ , cf. Benchmark Scheme 2, the capacity of the two-hop HD relay channel,  $C_{HD}$ , cf. Benchmark Scheme 3, and the rate achieved

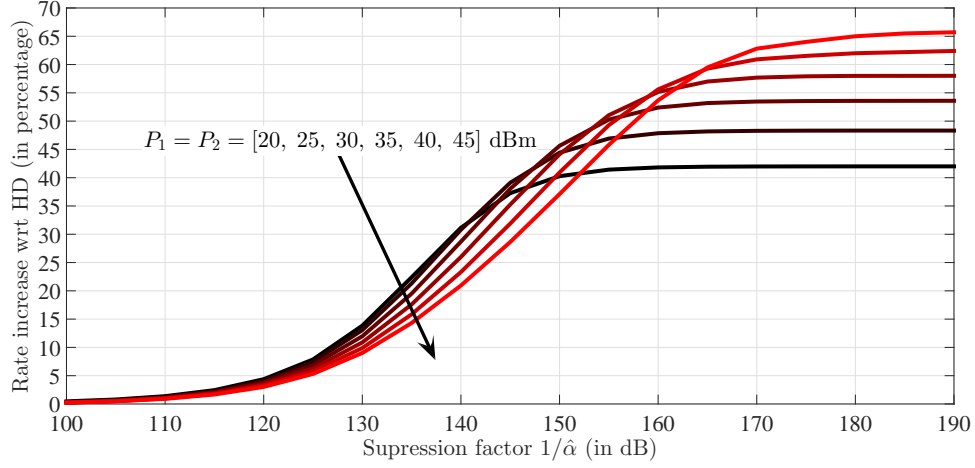


Fig. 4. Capacity gain of optimal FD relaying compared to optimal HD relaying as a function of the self-interference suppression factor,  $1/\hat{\alpha}$ , for different average transmit powers at source and relay  $P_1$  and  $P_2$ .

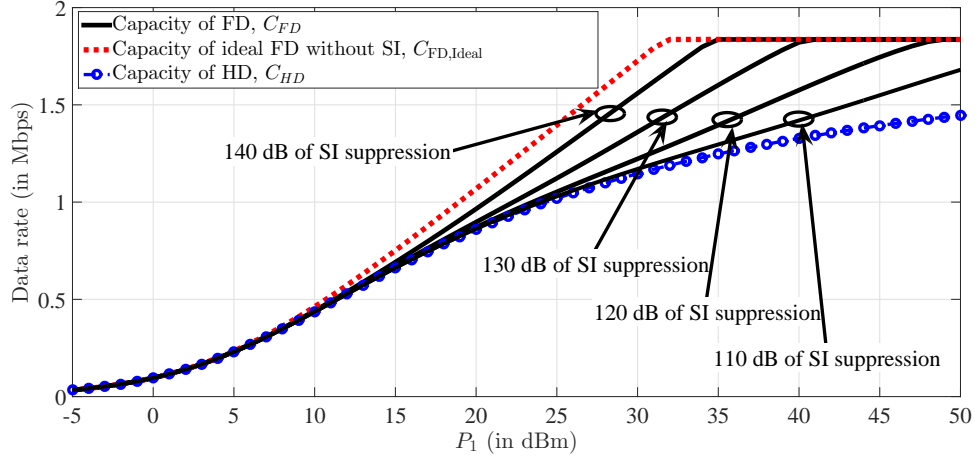


Fig. 5. Comparison of the derived capacity with the capacities achieved with ideal FD and optimal HD relaying as a function of the source's average transmit power  $P_1$  in dBm for a fixed transmit power at the relay of  $P_2 = 25$  dBm, and for different self-interference (SI) suppression factors,  $1/\hat{\alpha}$ .

with conventional HD relaying,  $R_{\text{HD,Conv}}$ , cf. Benchmark Scheme 4, for  $d_{SR} = d_{RD} = 500$  meter and a self-interference suppression factor,  $1/\hat{\alpha}$ , of 130 dB as a function of the average source and relay transmit powers  $P_1 = P_2$ . The figure shows that indeed the achievable rate with the suboptimal input distribution given in Section III-E,  $R_{\text{FD,B}}$ , is a tight lower bound on the capacity  $C_{\text{FD}}$ . Hence, this rate can be used for analytical analysis instead of the actual capacity rate, which is hard to analyze analytically. In addition, the figure shows that for  $P_1 = P_2 > 20$  dBm, the capacity  $C_{\text{FD}}$  becomes significantly larger than the rate achieved with conventional FD relaying,  $R_{\text{FD,Conv}}$ , where both source and relay transmit using Gaussian symbols without power allocation, cf. Benchmark Scheme 2. Also, for  $P_1 = P_2 > 20$  dBm, the proposed capacity-achieving coding

scheme achieves around 5 dB power gain compared to the two-hop HD relay channel, and around 10 dB power gain compared to conventional HD relaying. On the other hand, the capacity rate  $C_{\text{FD}}$  becomes significantly smaller than the capacity of the two-hop ideal FD relay channel without self-interference,  $C_{\text{FD,Ideal}}$ , when  $P_1$  and  $P_2$  exceed 20 dBm.

In Fig. 4, we show the capacity gain of the two-hop FD relay channel compared to the two-hop HD relay channel as a function of the self-interference suppression factor,  $1/\hat{\alpha}$ , for different average transmit powers at source and relay  $P_1 = P_2$  and  $d_{SR} = d_{RD} = 500$  meter. As can be seen from Fig. 4, for a self-interference suppression factor of 120 dB, we obtain only a 5 percent capacity increase of FD relaying compared to HD relaying. In contrast, for a self-interference suppression factor of 130 dB, we obtain around 10 to 15 percent increase in the capacity depending on the average transmit power. A 50 percent increase in the capacity is possible if  $P_1$  and  $P_2$  are larger than 25 dBm and the self-interference suppression factor is larger than 150 dB.

In Fig. 5, we compare the capacity of the considered FD relay channel,  $C_{\text{FD}}$ , with the capacities of the ideal FD relay channel without self-interference,  $C_{\text{FD,Ideal}}$ , and the HD relay channel,  $C_{\text{HD}}$ , for  $d_{SR} = 500$  meter,  $d_{RD} = 300$  meter, and  $P_2 = 25$  dBm as a function of the average transmit power at the source,  $P_1$ . Different self-interference suppression factors are considered. For this example, since the relay transmit power is fixed, the capacity of the relay-destination channel is also fixed to around 1.84 Mbps. As a result, the capacity of the considered relay channel cannot surpass 1.84 Mbps. In addition, it can be observed from Fig. 5 that the capacity of the considered FD relay channel,  $C_{\text{FD}}$ , is significantly larger than the capacity of the HD relay channel,  $C_{\text{HD}}$  when the transmit power at the source is larger than 30 dBm. For example, for 1.5 Mbps, the power gains are approximately 30 dB, 25 dB, 20 dB, and 15 dB compared to HD relaying for self-interference suppression factors of 140 dB, 130 dB, 120 dB, and 110 dB, respectively.

## V. CONCLUSION

We studied the capacity of the Gaussian two-hop FD relay channel with self-interference. For this channel, we considered the worst-case scenario when the self-interference is modeled as an i.i.d. Gaussian RV whose variance is proportional to the amplitude of the transmit symbols at the relay. We showed that the capacity is achieved by a zero-mean Gaussian input distribution at the source whose variance depends on the amplitude of the transmit symbols at the relay. On the other hand, the optimal input distribution at the relay is Gaussian only when the relay-destination link is the bottleneck link. Otherwise, the optimal input distribution at the relay is discrete. Our numerical results show that significant performance gains are achieved with the proposed capacity-achieving coding scheme compared to the achievable rates of conventional FD and HD relaying.

In addition, we proposed a suboptimal input distribution at the relay, which, for the presented numerical examples, achieves rates that are close to the capacity achieved with the optimal input distribution at the relay.

## APPENDIX

### A. Proof of Theorem 1

We first assume that  $p(x_2)$  is discrete. In addition, we assume that  $p(x_1|x_2)$  is a continuous distribution, which will turn out to be a valid assumption. Now, from (12), the corresponding maximization problem with respect to  $p(x_1|x_2)$  is given by

$$\begin{aligned} & \max_{p(x_1|x_2) \in \mathcal{P}} \sum_{x_2 \in \mathcal{X}_2} I(X_1; Y_1 | X_2 = x_2) p(x_2) \\ \text{Subject to C1: } & \sum_{x_2 \in \mathcal{X}_2} \left[ \int_{x_1} x_1^2 p(x_1|x_2) dx_1 \right] p(x_2) \leq P_1. \end{aligned} \quad (36)$$

Since  $I(X_1; Y_1 | X_2 = x_2)$  is the mutual information of a Gaussian AWGN channel with noise power  $\sigma_1^2 + \alpha x_2^2$ , cf. (6) the optimal distribution  $p(x_1|x_2)$  that maximizes  $I(X_1; Y_1 | X_2 = x_2)$  is the zero mean Gaussian distribution with variance  $P_1(x_2)$ . The variance  $P_1(x_2)$  has to satisfy constraint C1 in (36). Hence, to find the variance  $P_1(x_2)$ , we first substitute  $p(x_1|x_2)$  in (36) with the zero-mean Gaussian distribution with variance  $P_1(x_2)$ . Thereby, we obtain the following optimization problem

$$\begin{aligned} & \max_{P_1(x_2)} \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{P_1(x_2)}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2) \\ \text{Subject to C1: } & \sum_{x_2 \in \mathcal{X}_2} P_1(x_2) p(x_2) \leq P_1 \\ \text{C2: } & P_1(x_2) \geq 0, \forall x_2. \end{aligned} \quad (37)$$

Since (37) is a concave optimization problem, it can be solved in a straightforward manner using the Lagrangian method, which results in (13). In (13),  $x_{\text{th}}$  is a Lagrange multiplier which has to be set such that constraint C1 in (37) holds with equality. Inserting (13) into constraint C1 in (37), we obtain (14). Whereas, inserting  $P_1(x_2)$  in (13) into the objective function in (37), we obtain (15).

Following a similar procedure as above for the case when  $p(x_2)$  is assumed to be continuous, we arrive at the same solution for  $P_1(x_2)$  and  $\max_{p(x_1|x_2) \in \mathcal{P}} I(X_1; Y_1 | X_2)$  as in (13) and (15), respectively, but with the sums replaced by integrals. This concludes the proof.

### B. Proof of Theorem 2

Assuming  $p(x_2)$  is discrete, the corresponding capacity expression for  $p(x_1|x_2)$  given in Theorem 1 is given by

$$\begin{aligned}
 C &= \max_{p(x_2) \in \mathcal{P}} \min \left\{ \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2), I(X_2; Y_2) \right\} \\
 \text{Subject to C1: } &\sum_{x_2 \in \mathcal{X}_2} \alpha \max\{0, x_{\text{th}}^2 - x_2^2\} = P_1 \\
 \text{C2: } &\sum_{x_2 \in \mathcal{X}_2} x_2^2 p(x_2) \leq P_2
 \end{aligned} \tag{38}$$

Using its epigraph form, the optimization problem in (38) can be equivalently represented as

$$\begin{aligned}
 &\text{Maximize : } u \\
 &\quad p(x_2), u \\
 \text{Subject to C1 : } &u - \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2) \leq 0 \\
 \text{C2 : } &u - I(X_2; Y_2) \leq 0 \\
 \text{C3 : } &\sum_{x_2 \in \mathcal{X}_2} \alpha \max\{0, x_{\text{th}}^2 - x_2^2\} p(x_2) = P_1 \\
 \text{C4 : } &\sum_{x_2 \in \mathcal{X}_2} x_2^2 p(x_2) \leq P_2 \\
 \text{C5 : } &\sum_{x_2 \in \mathcal{X}_2} p(x_2) - 1 = 0.
 \end{aligned} \tag{39}$$

In the optimization problem (39), constraint C2 is convex with respect to  $p(x_2)$ , and constraints C1, C3, C4, and C5 are affine with respect to  $p(x_2)$ . Hence, the optimization problem in (39) is a concave optimization problem and can be solved using the Lagrangian method. The Lagrangian function of the optimization problem in (39) is given by

$$\begin{aligned}
 L &= u - \xi_1 \left( u - \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2) \right) - \xi_2 (u - I(X_2; Y_2)) \\
 &\quad - \lambda_1 \left( \sum_{x_2 \in \mathcal{X}_2} \alpha \max\{0, x_{\text{th}}^2 - x_2^2\} p(x_2) - P_1 \right) - \lambda_2 \left( \sum_{x_2 \in \mathcal{X}_2} x_2^2 p(x_2) - P_2 \right) - \nu \left( \sum_{x_2 \in \mathcal{X}_2} p(x_2) - 1 \right),
 \end{aligned} \tag{40}$$

where  $\xi_1$ ,  $\xi_2$ ,  $\lambda_1$ ,  $\lambda_2$ , and  $\nu$  are Lagrange multipliers corresponding to constraints C1, C2, C3, C4, and C5, respectively. Due to the KKT conditions, the following has to hold

$$\xi_1 \left( u - \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2) \right) = 0, \quad \xi_1 \geq 0 \quad (41a)$$

$$\xi_2 (u - I(X_2; Y_2)) = 0, \quad \xi_2 \geq 0 \quad (41b)$$

$$\lambda_1 \left( \sum_{x_2 \in \mathcal{X}_2} \alpha \max\{0, x_{\text{th}}^2 - x_2^2\} p(x_2) - P_1 \right) = 0 \quad (41c)$$

$$\lambda_2 \left( \sum_{x_2 \in \mathcal{X}_2} x_2^2 p(x_2) - P_2 \right) = 0, \quad \lambda_2 \geq 0, \quad (41d)$$

$$\nu \left( \sum_{x_2 \in \mathcal{X}_2} p(x_2) - 1 \right) = 0. \quad (41e)$$

Differentiating  $L$  with respect to  $u$ , we obtain that  $\xi_1 = 1 - \xi_2 = \xi$  has to hold, where  $0 \leq \xi \leq 1$ . Inserting this into (40), then differentiating with respect to  $p(x_2)$ , and equating the result to zero we obtain the following

$$\begin{aligned} & \xi \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) + (1 - \xi) I'(X_2; Y_2) \\ & - \lambda_1 \alpha \max\{0, x_{\text{th}}^2 - x_2^2\} - \lambda_2 x_2^2 - \nu = 0, \end{aligned} \quad (42)$$

where  $I'(X_2; Y_2) = \partial I(X_2; Y_2) / \partial p(x_2)$ . We note that there are three possible solutions for (42) depending on whether  $\xi = 1$ ,  $\xi = 0$ , or  $0 < \xi < 1$ , respectively. In the following, we analyze these three cases.

*Case 1:* Let us assume that  $\xi = 1$  holds. Then, from (41), we obtain that

$$u < I(X_2; Y_2) \text{ and } u = \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2), \quad (43)$$

which means that for the optimal  $p(x_2)$  the following holds

$$I(X_2; Y_2) \Big|_{p(x_2)=p^*(x_2)} > \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p^*(x_2). \quad (44)$$

The optimal  $p^*(x_2)$  in this case has to maximize the right hand side of (44), i.e.,

$$\sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2). \quad (45)$$

It turns out that the optimal  $p(x_2)$  which maximizes (45) is  $p^*(x_2) = \delta(x_2)$ , i.e., the relay is always silent and never transmits. However, if we insert  $p^*(x_2) = \delta(x_2)$  in  $I(X_2; Y_2)$  in (44), we obtain the following contradiction

$$I(X_2; Y_2) \Big|_{p(x_2)=\delta(x_2)} = 0 > \frac{1}{2} \log_2(1 + P_1/\sigma_1^2) > 0. \quad (46)$$



Hence,  $\xi = 1$  is not possible. The only remaining possibilities are  $\xi = 0$  and  $0 < \xi < 1$ .

*Case 2:* Let us assume that  $\xi = 0$  holds. Then, from (41), we obtain that

$$u = I(X_2; Y_2) \text{ and } u < \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2), \quad (47)$$

has to hold, which means that for the optimal  $p(x_2)$  the following holds

$$I(X_2; Y_2) \Big|_{p(x_2)=p^*(x_2)} < \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p^*(x_2). \quad (48)$$

The optimal  $p(x_2)$  in this case is the one which maximizes the left hand side of (48), i.e., maximizes  $I(X_2; Y_2)$ . Since the relay-destination link is an AWGN channel,  $I(X_2; Y_2)$  is maximized for  $p^*(x_2)$  being the zero-mean Gaussian distribution with variance  $P_2$ . As a result, the capacity is given by

$$I(X_2; Y_2) \Big|_{p(x_2)=p^*(x_2)} = \frac{1}{2} \log_2 \left( 1 + \frac{P_2}{\sigma_2^2} \right). \quad (49)$$

Hence, (49) is the capacity if and only if (iff) after substituting  $p^*(x_2)$  with the zero-mean Gaussian distribution with variance  $P_2$ , (48) holds, i.e., (16) holds.

*Case 3:* Let us assume that  $0 < \xi < 1$ . Then, from (41), we obtain that

$$u = I(X_2; Y_2) \text{ and } u = \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2), \quad (50)$$

which means that for the optimal  $p(x_2)$ , the following holds

$$I(X_2; Y_2) \Big|_{p(x_2)=p^*(x_2)} = \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p(x_2). \quad (51)$$

For  $0 < \xi < 1$ , we can find the optimal distribution  $p^*(x_2)$  as the solution of (42). To this end, we need to compute  $I'(X_2, Y_2)$ . Since for the AWGN channel,  $I(X_2; Y_2) = H(Y_2) - H(Y_2|X_2)$ , where  $H(Y_2|X_2) = \frac{1}{2} \log_2(2\pi e \sigma_2^2)$  hold, we obtain that  $I'(X_2; Y_2) = H'(Y_2)$ . On the other hand,  $H'(Y_2)$  for the AWGN channel is found as

$$H'(Y_2) = - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp \left( -\frac{(y_2 - x_2)^2}{2\sigma_2^2} \right) \log_2(p(y_2)) dy_2 - \frac{1}{\ln(2)}. \quad (52)$$

Inserting (52) into (42), we obtain

$$\begin{aligned} & \xi \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) - (1 - \xi) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp \left( -\frac{(y_2 - x_2)^2}{2\sigma_2^2} \right) \log_2(p(y_2)) dy_2 \\ & - (1 - \xi) \frac{1}{\ln(2)} - \lambda_1 \alpha \max\{0, x_{\text{th}}^2 - x_2^2\} - \lambda_2 x_2^2 - \nu = 0. \end{aligned} \quad (53)$$

Hence, the optimal  $p(x_2)$  has to produce a  $p(y_2)$  for which (53) holds. In Appendix C, we prove that (53) cannot hold if  $p(x_2)$  is a continuous distribution and that (53) can hold if  $p(x_2)$  is a discrete distribution since then it has to hold only for the discrete values  $x_2 \in \mathcal{X}_2$ .

*Remark 3:* Although we derived (42) assuming that  $p(x_2)$  is discrete, we would have arrived at the same result if we had assumed that  $p(x_2)$  is a continuous distribution. To do so, we first would have to replace the sums in the optimization problem in (39) with integrals with respect to  $x_2$ . Next, in order to obtain the stationary points of the corresponding Lagrangian function, instead of the ordinary derivative, we would have to take the functional derivative and equate it to zero. This again would have led to the identity in (42). Hence, the conclusions drawn from the Lagrangian and (42) are also valid when  $p(x_2)$  is a continuous distribution.

### C. Proof That $p(x_2)$ is Discrete when $0 < \xi < 1$

This proof is based on the proof for the discreteness of a distribution given in [33]. Furthermore, similar to [33], to simplify the derivation of the proof, we set  $\sigma_2^2 = 1$ .

First, we decompose the integral in (53) using Hermitian polynomials. To this end, we define

$$\log_2(p(y_2)) = \sum_{m=0}^{\infty} c_m H_m(y_2), \quad (54)$$

where the  $c_m$ ,  $\forall m$ , are constants and  $H_m(y_2)$ ,  $\forall m$ , are Hermitian polynomials, see [33]. Note that  $\ln(p(y_2))$  is square integrable with respect to  $e^{-\frac{y_2^2}{2}}$  and hence can be decomposed using a Fourier-Hermite series decomposition, see [33]. Then, the integral in (52) with  $\sigma_2^2 = 1$ , can be written as

$$\begin{aligned} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(y_2-x_2)^2}{2}} \log_2(p(y_2)) dy_2 &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{y_2^2}{2}} e^{(-\frac{x_2^2}{2} + x_2 y_2)} \log_2(p(y_2)) dy_2 \\ &\stackrel{(a)}{=} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{y_2^2}{2}} \sum_{n=0}^{\infty} H_n(y_2) \frac{x_2^n}{n!} \sum_{m=0}^{\infty} c_m H_m(y_2) dy_2 \stackrel{(b)}{=} \sum_{m=0}^{\infty} c_m x_2^m, \end{aligned} \quad (55)$$

where (a) is obtained by inserting (54) and using the generating function of Hermitian polynomials, given by

$$e^{(-\frac{t^2}{2} + tx)} = \sum_{m=0}^{\infty} H_m(x) \frac{t^m}{m!}. \quad (56)$$

Furthermore, (b) in (55) follows since Hermitian polynomials are orthogonal with respect to the weight function  $\omega(x) = e^{-\frac{x^2}{2}}$ , i.e.,

$$\int_{-\infty}^{\infty} H_m(x) H_n(x) \omega(x) dx = \begin{cases} m! \sqrt{2\pi} & \text{if } m = n \\ 0 & \text{otherwise} \end{cases} \quad (57)$$

holds. By inserting (55) into (53), we obtain

$$\begin{aligned} (1 - \xi) \sum_{m=0}^{\infty} c_m x_2^m &= \xi \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) - \lambda_1 \alpha \max\{0, x_{\text{th}}^2 - x_2^2\} - \lambda_2 x_2^2 \\ &\quad - (1 - \xi) \frac{1}{\ln(2)} - \nu. \end{aligned} \quad (58)$$

Now, we have two cases for  $|x_2|$ , one when  $|x_2| \geq x_{\text{th}}$  and the other one when  $|x_2| < x_{\text{th}}$ . Also, we have two cases for  $\lambda_2$ , one when  $\lambda_2 > 0$  (constraint C2 in (38) holds with equality) and the other are when  $\lambda_2 = 0$  (constraint C2 in (38) does not hold with equality). The resulting four cases are analyzed in the following.

*Case 1:* If  $|x_2| \geq x_{\text{th}}$  and  $\lambda_2 > 0$  hold, then (58) simplifies to

$$\sum_{m=0}^{\infty} c_m x_2^m = -\frac{\lambda_2}{1-\xi} x_2^2 - \frac{1}{\ln(2)} - \frac{\nu}{1-\xi}. \quad (59)$$

Comparing the exponents in (59), we obtain

$$c_0 = -\frac{1}{\ln(2)} - \frac{\nu}{1-\xi}; \quad c_1 = 0; \quad c_2 = \frac{\lambda_2}{1-\xi}; \quad c_n = 0, \quad \forall n > 2. \quad (60)$$

Inserting (60) into (54), we obtain  $p(y_2)$  as

$$p(y_2) = e^{\ln(2)(c_0 H_0(y_2) + c_2 H_2(y_2))} \stackrel{(a)}{=} e^{\ln(2)(c_0 - c_2)} e^{\ln(2)c_2 y_2^2}, \quad (61)$$

where (a) follows from  $H_0(y_2) = 1$  and  $H_2(y_2) = y_2^2 - 1$ . This solution for  $p(y_2)$  can be a valid probability density function only for  $c_2 < 0$ , and this yields a Gaussian distribution for  $p(y_2)$ . Now, for  $Y_2$  to be Gaussian distributed and  $Y_2 = X_2 + N_2$  to hold, where  $N_2$  is also Gaussian distributed, we  $p(x_2)$  also has to be Gaussian distributed. However, since the Gaussian distribution is unbounded in  $x_2$ , the Gaussian distribution  $p(x_2)$  cannot hold only in the domain  $|x_2| \geq x_{\text{th}}$  but has to hold in the entire domain  $|x_2| \leq \infty$ . Hence, we have to see whether a Gaussian  $p(x_2)$  is also optimal for  $|x_2| < x_{\text{th}}$ . If we obtain that  $p(x_2)$  is not Gaussian for  $|x_2| < x_{\text{th}}$ , then  $p(x_2)$  can only be discrete in the domain  $|x_2| \geq x_{\text{th}}$  for any  $x_{\text{th}} > 0$ .

*Case 2:* If  $|x_2| < x_{\text{th}}$  and  $\lambda_2 > 0$  hold, (58) simplifies to

$$\sum_{m=0}^{\infty} c_m x_2^m = \frac{\xi}{1-\xi} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha(x_{\text{th}}^2 - x_2^2)}{\sigma_1^2 + \alpha x_2^2} \right) - \frac{\lambda_1 \alpha + \lambda_2}{1-\xi} x_2^2 - \frac{1}{1-\xi} (1/\ln(2) + \nu + \lambda_1 \alpha x_{\text{th}}^2). \quad (62)$$

We now represent the  $\log_2(\cdot)$  function in (62) using a Taylor series expansion as

$$\log_2 \left( 1 + \frac{\alpha(x_{\text{th}}^2 - x_2^2)}{\sigma_1^2 + \alpha x_2^2} \right) = \sum_{n=0}^{\infty} (-1)^n a_n x_2^{2n}, \quad (63)$$

where  $a_n > 0$ ,  $\forall n$ , and the exact (positive) values of these coefficients are not important for this proof. Inserting (63) into (62), we obtain

$$\sum_{m=0}^{\infty} c_m x_2^m = \frac{\xi}{1-\xi} \frac{1}{2 \ln(2)} \sum_{n=0}^{\infty} (-1)^n a_n x_2^{2n} - \frac{\lambda_1 \alpha + \lambda_2}{1-\xi} x_2^2 - \frac{1}{1-\xi} (1/\ln(2) + \nu + \lambda_1 \alpha x_{\text{th}}^2). \quad (64)$$

Comparing the exponents on the left hand side and the right hand side of (64), we can find  $c_m$  as

$$c_m = \begin{cases} \frac{\xi}{1-\xi} \frac{1}{2 \ln(2)} a_0 - \frac{1}{1-\xi} (1/\ln(2) + \nu + \lambda_1 \alpha x_{\text{th}}^2) & \text{if } m = 0 \\ 0 & \text{if } m \text{ is odd} \\ \frac{\xi}{1-\xi} \frac{1}{2 \ln(2)} (-1) a_n x_2^2 - \frac{\lambda_1 \alpha + \lambda_2}{1-\xi} x_2^2 & \text{if } m = 2 \\ \frac{\xi}{1-\xi} \frac{1}{2 \ln(2)} (-1)^{m/2} a_{m/2} x_2^m & \text{if } m > 2 \text{ and } m \text{ is even} \end{cases} \quad (65)$$

Inserting (65) into (54), we obtain  $p(y_2)$  as

$$p(y_2) = e^{\ln(2) \sum_{m=0}^{\infty} c_{2m} H_{2m}(y_2)} = e^{\ln(2) \sum_{n=0}^{\infty} q_n y_2^{2n}} = \prod_{n=0}^{\infty} e^{\ln(2) q_n y_2^{2n}}, \quad (66)$$

where  $q_n$  are known nonzero constants. Now, since  $q_n > 0$  for some  $n \rightarrow \infty$ ,  $p(y_2)$  in (66) cannot be a valid distribution since  $p(y_2)$  becomes unbounded. As a result,  $p(x_2)$  has to be discrete in the domain  $|x_2| < x_{\text{th}}$ . Consequently,  $p(x_2)$  also has to be discrete in the domain  $|x_2| \geq x_{\text{th}}$ . This concludes the proof for the case when  $\lambda_2 > 0$ . Following a similar procedure for  $\lambda_2 = 0$  as for the case when  $\lambda_2 > 0$ , we obtain that again  $p(x_2)$  has to be discrete in the entire domain for  $x_2$ .

On the other hand,  $p^*(x_2)$  has to be symmetrical with respect to  $x_2 = 0$ . To prove this, assume that we have an unsymmetrical  $p(x_2)$ , denoted by  $p_u(x_2)$ , with only one unsymmetrical mass point  $x_{2u}$  which has probability  $p_{2u}$ . Now, let us construct a new, symmetrical  $p(x_2)$ , denoted by  $p_s(x_2)$ , by making  $p_u(x_2)$  symmetrical. In particular, in  $p_u(x_2)$ , we first reduce the probability of the mass point  $x_{2u}$  to  $p_{2u}/2$ . Next, we add the mass point  $-x_{2u}$  to  $p_u(x_2)$  and set its probability to  $p_{2u}/2$ . Now, it is clear that the average power of the relay is identical for both  $p_u(x_2)$  and  $p_s(x_2)$ . On the other hand, by making  $p(x_2)$  symmetrical, we have increased the entropy of  $X_2$ , i.e.,  $H(X_2)|_{p(x_2)=p_u(x_2)} \leq H(X_2)|_{p(x_2)=p_s(x_2)}$  holds. Consequently, we have increased the differential entropy of  $Y_2$ , i.e.,  $h(Y_2)|_{p(x_2)=p_u(x_2)} \leq h(Y_2)|_{p(x_2)=p_s(x_2)}$  holds. Now, since for the AWGN channel  $h(Y_2|X_2)$  is independent of  $p(x_2)$ , it follows that  $I(X_2; Y_2)|_{p(x_2)=p_u(x_2)} \leq I(X_2; Y_2)|_{p(x_2)=p_s(x_2)}$  holds. This concludes the proof for the symmetry of  $p^*(x_2)$ .

#### D. Proof of Lemma 1

Here, we only prove the non-trivial case when (16) does not hold. The trivial case is identical to the case without self-interference and its achievability is shown [2].

Let us assume that condition (16) does not hold. Then, according to Theorem 1,  $p(x_2)$  is discrete and the capacity  $C$  is given in (19). Moreover, for the considered coding scheme,  $R$  satisfies the

following

$$\begin{aligned}
R < C &= \max_{p(x_1|x_2)} I(X_1; Y_1|X_2) = \sum_{x_2 \in \mathcal{X}_2} \max_{p(x_1|x_2)} I(X_1; Y_1|X_2 = x_2) p^*(x_2) \\
&\stackrel{(a)}{=} \sum_{\substack{x_2 \in \mathcal{X}_2 \\ |x_2| < x_{\text{th}}}} \max_{p(x_1|x_2)} I(X_1; Y_1|X_2 = x_2) p^*(x_2) \stackrel{(b)}{\leq} \sum_{\substack{x_2 \in \mathcal{X}_2 \\ |x_2| < x_{\text{th}}}} \max_{p(x_1|x_2)} I(X_1; Y_1|X_2 = 0) p^*(x_2) \\
&= \max_{p(x_1|x_2)} I(X_1; Y_1|X_2 = 0) \sum_{\substack{x_2 \in \mathcal{X}_2 \\ |x_2| < x_{\text{th}}}} p^*(x_2) = \max_{p(x_1|x_2)} I(X_1; Y_1|X_2 = 0) p_T \\
&\stackrel{(c)}{=} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha x_{\text{th}}^2}{\sigma_1^2} \right) p_T, \tag{67}
\end{aligned}$$

where (a) follows since for the considered coding scheme the source is silent when  $|x_2| \geq x_{\text{th}}$  and as a result  $I(X_1; Y_1|X_2 = x_2) = 0$  for  $|x_2| \geq x_{\text{th}}$ , (b) follows since, for the considered relay channel,  $I(X_1; Y_1|X_2 = x_2)$  is maximized for  $x_2 = 0$ , because in that case there is no self-interference at the relay, and (c) follows from (15).

Now, note that for the considered coding scheme in time slot 1, the source-relay channel can be seen as an AWGN channel with a fixed channel gain  $\sqrt{P_1(x_2 = 0)} = \sqrt{\alpha} x_{\text{th}}$  and AWGN with variance  $\sigma_1^2$  which is used  $k p_T$  times. Hence, any codeword selected uniformly from a codebook comprised of  $2^{kR}$  Gaussian distributed codewords, where each codeword is comprised of  $k p_T$  symbols, with  $k \rightarrow \infty$  and  $R$  satisfying

$$kR/(k p_T) < \frac{1}{2} \log_2 \left( 1 + \frac{\alpha x_{\text{th}}^2}{\sigma_1^2} \right), \tag{68}$$

can be successfully decoded at the relay using a jointly-typical decoder, see [29]. Noting that the proposed coding scheme satisfies the properties outlined above, we can conclude that the codeword transmitted in time slot 1 can be decoded successfully at the relay.

### E. Proof of Lemma 2

Again, we only prove the non-trivial case when (16) does not hold.

In time slot  $b$ , for  $2 \leq b \leq N$ , the source-relay channel can be seen equivalently as an AWGN channel with states  $X_2$ , where a different state  $X_2 = x_2$  produces a different channel gain and a different noise variance. In particular, for channel state  $X_2 = x_2$ , the channel gain of the equivalent AWGN channel is  $\sqrt{P_1(x_2)}$  and the variance of the AWGN is  $\sigma_1^2 + \alpha x_2^2$ . Moreover, for this equivalent AWGN channel with states the source has to transmit unit-variance symbols in order for the average power constraint of the original source-relay channel, given by  $E\{X_1^2\} \leq P_1$ , to be satisfied. Furthermore, for the equivalent AWGN channel with states, note that both source (i.e., transmitter) and relay (i.e., receiver) have channel state information (CSI) in each channel use

and thereby know that the channel gain and the noise variance in channel use  $j$  will be  $\sqrt{P_1(x_{2,j})}$  and  $\sigma_1^2 + \alpha x_{2,j}^2$ , respectively. Now, instead of deriving a capacity-achieving coding scheme for the original source-relay channel, we can find equivalently a capacity-achieving coding scheme for the equivalent AWGN channel<sup>4</sup> with states. To this end, we will first find the capacity of an “auxiliary AWGN channel”, using results which are already available in the literature. Then, we will modify the capacity-achieving coding scheme of the “auxiliary AWGN channel” in order to obtain a capacity-achieving coding scheme for the equivalent AWGN channel with states.

The “auxiliary AWGN channel” is identical to the equivalent AWGN channel but without CSI at the source (i.e., transmitter). The channel code that achieves the capacity of the “auxiliary AWGN channel” in  $k \rightarrow \infty$  channel uses is the following, see [34], [35] for proof. The codebook is comprised of  $2^{kR}$  codewords, where each codeword is comprised of  $k$  symbols and each symbol is generated independently according to the zero-mean unit-variance Gaussian distribution. Moreover, the parameter  $R$  of the channel code has to satisfy

$$\begin{aligned}
R &< \max_{\substack{p(x'_1|x_2) \\ E\{X_1'^2\}=1}} I(X'_1; Y_2 | X_2) \Big|_{p(x_2)=p^*(x_2)} = \sum_{x_2 \in \mathcal{X}_2} \max_{\substack{p(x'_1|x_2) \\ E\{X_1'^2\}=1}} I(X'_1; Y_1 | X_2 = x_2) p^*(x_2) \\
&\stackrel{(a)}{=} \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{P_1(x_2)}{\sigma_1^2 + \alpha x_2^2} \right) p^*(x_2), \\
&\stackrel{(b)}{=} \sum_{x_2 \in \mathcal{X}_2} \frac{1}{2} \log_2 \left( 1 + \frac{\alpha \max\{0, x_{\text{th}}^2 - x_2^2\}}{\sigma_1^2 + \alpha x_2^2} \right) p^*(x_2),
\end{aligned} \tag{69}$$

where  $X'_1$  is the input at the source of the “auxiliary AWGN channel”, (a) follows due to the unit-variance constraint  $E\{X_1'^2\} = 1$  and since for each state  $X_2 = x_2$  the channel is AWGN with channel gain  $\sqrt{P_1(x_{2,j})}$  and noise variance  $\sigma_1^2 + \alpha x_{2,j}^2$ , and (b) follows from (13). Any codeword selected uniformly from this codebook and transmitted in  $k$  channel uses can be successfully decoded at the relay (i.e., receiver) using a jointly typical decoder, see [34], [35], [29]. Now, for the “auxiliary AWGN channel” note that the source transmits a symbol during all  $k$  channel uses. Hence, the source transmits a symbol during channel uses for which the channel gain is zero, i.e.,  $\sqrt{P_1(x_2)} = 0$  holds. Obviously, the symbols transmitted when  $\sqrt{P_1(x_2)} = 0$  do not reach the relay due to the zero channel gain, i.e., the relay receives only noise during these channel uses.

Now, for the equivalent AWGN channel we can use the same coding scheme as for the “auxiliary AWGN channel”, but, since in this case the source has CSI, the source can choose not to transmit

<sup>4</sup>The capacity-achieving coding scheme of the original source-relay channel can be obtained straightforwardly from the equivalent AWGN channel with states. In particular, the only modification is that the source has to multiply the transmitted symbol in channel use  $j$  by  $\sqrt{P_1(x_{2,j})}$ .

during a channel use for which  $\sqrt{P_1(x_2)} = 0$  holds. Moreover, since the source has knowledge that  $\sqrt{P_1(x_{2,j})} > 0$  holds in a  $p_T$  fraction out of the  $k$  channel uses, the source can reduce the length of the codewords from  $k$  to  $k p_T$ . Thereby, the channel code for the equivalent AWGN channel has a codebook comprised of  $2^{kR}$  Gaussian distributed codewords, where each codeword is comprised of  $k p_T$  symbols. Moreover, for the equivalent AWGN channel, the source is silent for states for which  $\sqrt{P_1(x_2)} = 0$  holds, i.e.,  $|x_2| \geq x_{th}$  holds, and transmits a symbol from the selected codeword only when  $\sqrt{P_1(x_2)} > 0$ , i.e.,  $|x_2| < x_{th}$  holds, which is exactly our proposed scheme. Hence, for our proposed scheme, we can conclude that the codeword transmitted in time slot  $b$ , for  $2 \leq b \leq N$ , can be decoded successfully at the relay.

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